HOMEWORK 1

Students enrolled in this course are required to submit their solutions for this homework before 11:59 am on 11 February 2026, using one of the following methods.

(1) Send me an email with the solution attached.
(2) Submit a hard copy to me in person before class.

Please write your solutions clearly and neatly, and include sufficient explanations so that your reasoning is easy to follow.

Problem 1. Let G be a topological group and $H \subseteq G$ be a closed normal subgroup. Show that G/H, equipped with the quotient topology, is a topological group.

Problem 2. Let G be a Lie group and (H', φ) be an embedded Lie subgroup of G, i.e. $\varphi : H' \to G$ is an embedding. Let $H := \varphi(H')$.

(1) Let $\overline{H}$ be the closure of H in G. Show that $\overline{H}$ is a subgroup of G.
(2) Show that each coset Hx. $x \in \overline{H}$, is open and dense in $\overline{H}$.
(3) Show that $\overline{H} = H$. Thus, every embedded Lie subgroup is a closed Lie subgroup.

Problem 3. Let X be a topological space. Prove the following.

(1) If X is path–connected, then X is connected.
(2) Assume X is a topological manifold. If X is connected, then X is path–connected.

Problem 4. Let G be a connected Lie group. Show that every discrete normal subgroup N of G is contained in the center of G.

Problem 5. Let G and H be Lie groups.

(1) Show that there is a natural isomorphism
$$T_{(1,1)}(G \times H) \cong T_1G \oplus T_1H.$$
(2) Let $m : G \times G \to G$ be the multiplication map on G. Show that the differential of m at $(1, 1) \in G \times G$ is given by
$$\begin{array}{rccc} T_{(1,1)}m : & T_{(1,1)}(G \times G) \cong T_1G \oplus T_1G & \longrightarrow & T_1G \\ & (X, Y) & \longmapsto & X + Y. \end{array}$$
(3) Let $\iota : G \to G$ be the inversion map. Show that the differential of ι at $1 \in G$ is given by
$$\begin{array}{rccc} T_1\iota : & T_1G & \longrightarrow & T_1G \\ & X & \longmapsto & -X. \end{array}$$

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